TITLES and ABSTRACTS

Robert Berman

Title: **Canonical metrics, random point processes and tropicalization**

Abstract: In this talk I will present a survey of the connections between canonical metrics and random point processes on a complex algebraic variety X. When the variety X has positive Kodaira dimension, this leads to a probabilistic construction of the canonical metric on X introduced by Tsuji and Song-Tian (coinciding with the Kähler-Einstien metric when X is of general type). In the opposite setting of Fano varieties this suggests a probalistic analog of the Yau-Tian-Donaldson conjecture. The probabilistic version of the conjecture is open, in general. But, as shown in a recent joint work with Magnus Önnheim, for toric X the "tropicalized" version of the conjecture does hold and involves discrete optimal transport theory.

Bo Berndtsson

Title: **The curvature of (higher) direct images**

Abstract: I will first discuss some earlier work on the curvature of direct images of adjoint line bundles under a smooth proper fibration, or more generally a surjective map and (maybe) some of its applications. Then I will present a general formula for the curvature of higher direct images. This contains as a special case (part of) the earlier work, and also generalizes results of Siu and Schumacher. (The second part is based on joint work with Mihai Paun and Xu Wang.)

Damian Brotbek

Title: **On the hyperbolicity of general hypersurfaces**

Abstract: A smooth projective variety over the complex numbers is said to be (Brody) hyperbolic if it doesn't contain any entire curve. Kobayashi conjectured in the 70's that general hypersurfaces of sufficiently large degree in $\mathbb{P}^N$ are hyperbolic. This conjecture was only recently proved by Siu.

The purpose of this talk is to present a new proof of this conjecture. The main idea of the proof, based on the theory of jet differential equations, is to establish that a stronger property, open in the Zariski topology, is satisfied for suitable deformations of Fermat type hypersurfaces.

Frédéric Campana

Title: **Birational stability of the orbifold cotangent bundle**

Abstract: We show that a foliation on a projective complex manifold is algebraic with rationally connected (closure of) leaves exactly when its minimal slope with respect to some movable class is positive. This extends and strengthens former classical results by Y. Miyaoka and Bogomolov-Mc Quillan. Applications to foliations, hyperbolicity (a converse to a result of JP. Demailly) and moduli will be mentioned.This is a joint work with Mihai Paun, partly based on a former joint work with T. Peternell.

Tristan Collins

Title: **The Complexified Kähler Cone**

Abstract: I will discuss a proposal for a definition of the complexified Kähler cone, motivated by the SYZ program for mirror symmetry, which involves solving a "complexified" version of the Monge-Ampere equation. In particular, I will discuss a conjectural characterization of the complexified Kähler cone in terms of algebro-geometric data, inspired by the Demailly-Paun characterization of the Kähler cone of compact Kähler manifolds, and Bridgeland stability conditions. Time permitting, I will discuss some further analogies, including complex analogs of the Khovanskii-Teissier inequalities, inspired by Demailly's proof of these inequalities via the complex Monge-Ampere equation. This talk is based on joint works with A. Jacob, S.-T. Yau and G. Szekelyhidi.

Tamás Darvas

Title: **Complex Monge-Ampère equations with prescribed singularity type**

Abstract: Given a Kahler manifold (X, ω), finding smooth solutions to the equation $(\omega + i\partial\bar{\partial}u)^n = f\omega^n$ goes back to Yau's solution of the Calabi conjecture in the seventies. In joint work with E. Di Nezza and C.H. Lu, we proposed to solve this same equation with the added constraint that $u \in \mathrm{PSH}(X, \omega)$ has prescribed singularity type. As it turns out, this problem is well posed only for a certain class of (model) singularity types that we characterize, and we also solve the corresponding equation. Our results extend to the case of big cohomology classes as well.

Simon Donaldson

Title: **Boundary value problems for G_2 structures**

Abstract: In the lecture we consider the existence of G_2 structures on 7-manifolds with boundary, with prescribed data on the boundary. In the first part we will review general background and theory, including Hitchin's variational approach. We will then discuss in more detail reductions of the problem in the presence of symmetry and in "adiabatic limits", and connections with real and complex Monge-Ampère equations.

Charles Favre

Title: **Degeneration of measures of maximal entropy**

Abstract: Consider any meromorphic family of endomorphisms of the complex projective plane parameterized by the punctured unit disk. We shall explain how to describe the behaviour of their measures of maximal entropy when one approaches the central fiber. This generalizes works by Demarco and Faber.

Vincent Guedj

Title: **Quasi-psh envelopes and supersolutions**

Abstract: Using and extending an approximation process due to Berman, we show that the quasi-psh envelope of a viscosity super-solution is a pluripotential super-solution of a given complex Monge-Ampère equation. We apply these ideas to Kahler-Einstein geometry (joint work with H.C.Lu and A.Zeriahi).

Sławomir Kołodziej

Title: **Monge-Ampère and Hessian equations on compact Hermitian manifolds**

Abstract: Let (X, ω) be a compact Hermitian manifold of complex dimension n. I shall discuss some recent results concerning weak solutions to the complex Monge-Ampère equation and, the more general, complex Hessian equation

$$(\omega + dd^c\varphi)^k \wedge \omega^{n-k} = cf\omega^n,$$

(where $0 \leq f$ belongs to some L^p space) including existence, stability and Hölder continuity. They were obtained in collaboration with Sławomir Dinew and Cuong Ngoc Nguyen. I would like to highlight interesting open problems.

Ngaiming Mok

Title: **Linearly saturated subvarieties on uniruled projective manifolds**

Abstract: Let Z and X be uniruled projective manifolds of Picard number 1 such that the respective variety of minimal rational tangents (VMRT) at a general point satisfies a nondegeneracy condition on the second fundamental form. In 2001 Hwang and Mok established the equidimensional Cartan-Fubini extension principle, according to which a germ of VMRT-preserving holomorphic map $f : (Z, z_0) \to (X, x_0)$ must necessarily extend to a biholomorphism $F : Z \to X$. In 2010, Hong

and Mok extended this to the nonequidimensional case for germs of holomorphic immersions between uniruled projective manifolds, allowing $\dim(Z) < \dim(X)$, by proving that f must necessarily extend to a rational map provided that a certain relative version of the nondegeneracy condition on the second fundamental form is satisfied. Very recently, Mok and Zhang developed the theory of geometric substructures by considering germs of complex submanifolds of $(S, x_0) \hookrightarrow (X, x_0)$ and introducing geometric substructures on S by taking intersections of the VMRTs of X with projectivized tangent spaces of S. We introduced a new relative nondegeneracy condition related to the second fundamental form and proved the extendibility of the germ (S, x_0) to a projective subvariety $Y \subset X$ under the assumption that X is uniruled by lines, i.e., by rational curves whose homology classes are the positive generator of $H^2(X, \mathbb{Z}) \cong \mathbb{Z}$. We achieved this by recovering Y as the image under a tautological map of a certain universal family of chains of minimal rational curves. The existence of the latter family is obtained by means of analytic continuation of the Thullen type for germs of holomorphic substructures.

Mircea Mustata

Title: **Hodge ideals**

Abstract: I will discuss certain invariants of singularities, the Hodge ideals, that are defined in the context of Saito's theory of mixed Hodge modules. They can be considered as higher order analogues of the multiplier ideals, invariants that have had a lot of applications in complex geometry. I will describe some general properties of Hodge ideals and some applications. This is joint work with Mihnea Popa.

Takeo Ohsawa

Title: **On the local pseudoconvexity of certain families of $\mathbb{C}$**

Abstract: For a class of weakly 1-complete $\mathbb{C}-$ bundles over compact Riemann surfaces, for which canonical plurisubharmonic exhaustion functions on the total spaces are known, some cases are described where such functions can be extended to a plurisubharmonic exhaustion function on analytic families of the $\mathbb{C}-$ bundles. The nonextendable cases are also discussed.

Yum-Tong Siu

Title: **Differential Relations of Multiplier Ideal Sheaves in Regularity Problems of PDE**

Abstract: Multiplier ideal sheaves are introduced to measure the location and the jet order of the failure of a priori estimates in regularity problems of partial differential equations. The key tool is differential relations of multiplier ideal sheaves involving differentiation in the good directions, along which a priori estimates hold. We will discuss the background, motivation, known results, open problems and approaches to them.

Valentino Tosatti

Title: $C^{1,1}$ **estimates for complex Monge-Ampère equations**

Abstract: I will discuss a method that we recently introduced in collaboration with Chu and Weinkove which gives interior $C^{1,1}$ estimates for the non-degenerate complex Monge-Ampère equation on compact Kähler manifolds (possibly with boundary). The method is sufficiently robust to also give $C^{1,1}$ regularity of geodesic segments in the space of Kähler metrics (thus resolving a long-standing problem originating from the work of Chen), of quasi-psh envelopes in Kähler as well as nef and big classes (solving a conjecture of Berman), and of geodesic rays that arise from test configurations (improving results of Phong and Sturm), and it even applies to the almost-complex case.

Claire Voisin

Title: **Cubic fourfolds, hyper-Kähler manifolds and their degenerations**

Abstract: There at least three families of hyper-Kähler manifolds built from cubic fourfolds, the most recently discovered one being the compactified intermediate Jacobian fibrations I constructed with Laza and Saccà. In a joint work with Kollár, Laza and Saccà, we provide an easy way to compute their deformation types, by proving that if the central fiber of a degeneration of hyper-Kähler manifolds has one component which is not uniruled, then after base-change the family becomes fiberwise birational to a family of smooth hyper-Kähler manifolds.

Shing-Tung Yau

Title: **Existence of complete Kähler-Einstein metric with negative scalar curvature**

Abstract: This is a talk about my works with Damin Wu concerning those manifolds with negative holomorphic sectional curvature. I shall describe our theorem that such manifold must have negative first Chern class.